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带路径圈根乘积细分线图基于度的指数

沙夫卡特·阿里
（哈吉大学 数学系，拉合尔 54000）

摘要：借助拓扑指数，有利于对有关图形族的拓扑数值量的理解，也可以更好地了解相关的图族. 为探讨确定带路径的循环根生成细分的线图基于度的拓扑指数，利用边分解和图结构分析方法，计算了带下划线的图族的调和指数、Randic 类型指数、对称除法指数、原子键连通指数、调和指数、对称分割指数、几何算术指数、广义逆 Randic 指数和逆和指数等.

关键词：线图；细分图；拓扑指数；萨格勒布指数
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Degree-based Indices of Line Graph of the Subdivision of Rooted Product of Cycles with Paths

SHAFQAT Ali
（Department of Mathematics, Minhaj University, Lahore, Pakistan　54000）

Abstract: With the help of topological index, it's helpful to understand the numerical quantities of concerned family of graphs. The aim of the present report is to determine the degree-based topological indices of the line graph of the subdivision of rooted product of cycles with paths. By means of edge dividing and graph structure analysis, we computed Harmonic index, Randic type indices, symmetric division index, atomic-bond-connectivity index, harmonic-index, symmetric division index, geometric-arithematic index, generalized reverse-Randic index, and inverse sum index of underlined family of graphs.

Key words: line graph; subdivision graph; topological index; Zagreb index

1　Background Knowledge

The concept of rooted product graph was introduced in 1978 by Godsil and McKay[1]. Given a graph G of order $n(G)$ and a graph H with root vertex v, the rooted product graph $G\{H\}$ is defined as the graph obtained from G and H by taking one copy of G and $n(G)$ copies of H and identifying the i-th vertex of G with the root vertex v in the ith copy of H for every $i \in \{1, 2, \cdots, n(G)\}$. If H or G is a trivial graph, then $G\{H\}$ is equal to G or H, respectively. In this paper we aim to study the rooted product of cycles with paths.

The line graph of an undirected graph G is another graph $L(G)$ that represents the adjacencies between edges of G[2].

The real number attached with the graph of chemical structure is known ad topological index. The theory of topological indices begun in 1947, when Wiener index was introduced[3]. After this huge amount of topological indices are introduced to study graphs, see for example[4]. Here we study some of them. From now to on word, we consider G to be connected and simple graph. Now we give some definitions of topological indices that can be found in [5 - 6].

The Symmetric-division-index of G is:

$$SDI(G) = \sum \left\{\frac{\min(d_u, d_v)}{\max(d_u, d_v)} + \frac{\max(d_u, d_v)}{\min(d_u, d_v)}\right\}.$$

The Harmonic-index of G is:

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作者简介：沙夫卡特·阿里，男，巴基斯坦人，教授，博士，主要从事图论研究.

$$H(G)=\sum\frac{2}{d_u+d_v}.$$

The Inverse-Sum index of G is:

$$I(G)=\sum_{uv\in E(G)}\frac{d_u\times d_v}{d_u+d_v}.$$

The generalized Randic index of G is:

$$R_\alpha(G)=\sum\frac{1}{(d_u\times d_v)^\alpha}.$$

The generalized reverse-Randic index of G is:

$$RR_\alpha(G)=\sum(d_u\times d_v)^\alpha.$$

The atomic-bond-connectivity index of G is:

$$ABC(G)=\sum\sqrt{\frac{d_u+d_v-2}{d_u\times d_v}}.$$

The geometric-arithematic index of G is:

$$GA(G)=\sum\frac{2\sqrt{d_u\times d_v}}{(d_u+d_v)}.$$

The modified-Randic index of G is:

$$R'(G)=\sum\frac{1}{\max(d_u,d_v)}.$$

In this paper, we computed all above defined degree-based topological indices for the line graph of the subdivision graph of rooted product of cycles with paths.

2 Main Results

In this section, we will present our main computational results.

2.1 Line Graph of the Subdivision Graph of $C_n\{P_k\}$ for $k>1$

The line graph of rooted product of cycle and path for $k>1$ is presented in Figure 1.

The edge partition for $C_n\{P_k\}$ is presented in Table 1.

Table 1 Edge Partition of $E(C_n\{P_k\})$

(d_u,d_v) Where $uv\in E(C_n\{P_k\})$ for $k>1$	Number of Edges
(1,2)	n
(2,2)	$2nk-3n$
(2,3)	n
(3,3)	$4n$

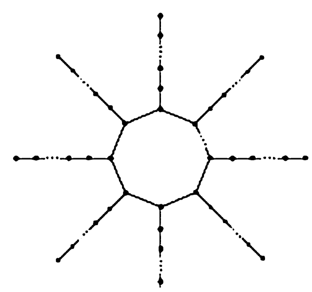

Figure 1 Line Graph of the Subdivision Graph of $C_n\{P_k\}$

Theorem 1 For $C_n\{P_k\}$ for $k>1$, the Harmonic index is:

$$H(C_n\{P_k\})=n(k+\frac{9}{10}).$$

Proof Using the edge partition given in Table 1, we have following computation of Harmonic index:

$$\begin{aligned}H(C_n\{P_k\})&=\sum_{uv\in E(C_n\{P_k\})}\frac{2}{d_u+d_v}\\&=\sum_{uv\in E_1(C_n\{P_k\})}\frac{2}{1+2}+\sum_{uv\in E_2(C_n\{P_k\})}\frac{2}{2+2}+\sum_{uv\in E_3(C_n\{P_k\})}\frac{2}{2+3}+\sum_{uv\in E_4(C_n\{P_k\})}\frac{2}{3+3}\\&=\frac{2}{3}|E_1(C_n\{P_k\})|+\frac{1}{2}|E_2(C_n\{P_k\})|+\frac{2}{5}|E_3(C_n\{P_k\})|+\frac{1}{3}|E_4(C_n\{P_k\})|\\&=(n)\frac{2}{3}+(2nk-3n)\frac{1}{2}+(n)\frac{2}{5}+(4n)\frac{1}{3}\end{aligned}$$

$$= n\left(k + \frac{9}{10}\right).$$

Theorem 2　For $C_n\{P_k\}$ for $k > 1$, the inverse sum index is:

$$I(C_n\{P_k\}) = n\left(2k + \frac{73}{15}\right).$$

Proof　Using the edge partition given in Table 1, we have following computation of inverse-sum index:

$$\begin{aligned}
I(C_n\{P_k\}) &= \sum_{uv \in E(C_n\{P_k\})} \frac{d_u \times d_v}{d_u + d_v} \\
&= \sum_{uv \in E_1(C_n\{P_k\})} \frac{1 \times 2}{1 + 2} + \sum_{uv \in E_2(C_n\{P_k\})} \frac{2 \times 2}{2 + 2} + \sum_{uv \in E_3(C_n\{P_k\})} \frac{2 \times 3}{2 + 3} + \sum_{uv \in E_4(C_n\{P_k\})} \frac{3 \times 3}{3 + 3} \\
&= \frac{2}{3}|E_1(C_n\{P_k\})| + 1|E_2(C_n\{P_k\})| + \frac{6}{5}|E_3(C_n\{P_k\})| + \frac{3}{2}|E_4(C_n\{P_k\})| \\
&= \frac{2}{3}(n) + 1(2nk - 3n) + \frac{6}{5}(n) + \frac{3}{2}(4n) \\
&= n\left(2k + \frac{73}{15}\right).
\end{aligned}$$

Theorem 3　For $C_n\{P_k\}$ for $k > 1$, the generalized Randic index is:

$$R_\alpha(C_n\{P_k\}) = n\left(\frac{1}{2^\alpha} + \frac{1}{4^\alpha}(2k - 3) + \frac{1}{6^\alpha} + \frac{4}{9^\alpha}\right).$$

Proof　Using the edge partition given in Table 1, we have following computation of generalized Randic index:

$$\begin{aligned}
R_\alpha(C_n\{P_k\}) &= \sum_{uv \in E(C_n\{P_k\})} \frac{1}{(d_u \times d_v)^\alpha} \\
&= \sum_{uv \in E_1(C_n\{P_k\})} \frac{1}{(1 \times 2)^\alpha} + \sum_{uv \in E_2(C_n\{P_k\})} \frac{1}{(2 \times 2)^\alpha} + \sum_{uv \in E_3(C_n\{P_k\})} \frac{1}{(2 \times 3)^\alpha} + \sum_{uv \in E_4(C_n\{P_k\})} \frac{1}{(3 \times 3)^\alpha} \\
&= \frac{1}{2^\alpha}|E_1(C_n\{P_k\})| + \frac{1}{4^\alpha}|E_2(C_n\{P_k\})| + \frac{1}{6^\alpha}|E_3(C_n\{P_k\})| + \frac{1}{9^\alpha}|E_4(C_n\{P_k\})| \\
&= \frac{1}{2^\alpha}(n) + \frac{1}{4^\alpha}(2nk - 3n) + \frac{1}{6^\alpha}(n) + \frac{1}{9^\alpha}(4n) \\
&= n\left(\frac{1}{2^\alpha} + \frac{1}{4^\alpha}(2k - 3) + \frac{1}{6^\alpha} + \frac{4}{9^\alpha}\right).
\end{aligned}$$

Theorem 4　For $C_n\{P_k\}$ for $k > 1$, the inverse generalized Randic index is:

$$RR_\alpha(C_n\{P_k\}) = n(2^\alpha + (2k - 3)4^\alpha + 6^\alpha + 4.9^\alpha).$$

Proof　Using the edge partition given in Table 1, we have following computation of the inverse generalized Randic index:

$$\begin{aligned}
RR_\alpha(C_n\{P_k\}) &= \sum_{uv \in E(C_n\{P_k\})} (d_u \times d_v)^\alpha \\
&= \sum_{uv \in E_1(C_n\{P_k\})} (1 \times 2)^\alpha + \sum_{uv \in E_2(C_n\{P_k\})} (2 \times 2)^\alpha + \sum_{uv \in E_3(C_n\{P_k\})} (2 \times 3)^\alpha + \sum_{uv \in E_4(C_n\{P_k\})} (3 \times 3)^\alpha \\
&= 2^\alpha|E_1(C_n\{P_k\})| + 4^\alpha|E_2(C_n\{P_k\})| + 6^\alpha|E_3(C_n\{P_k\})| + 9^\alpha|E_4(C_n\{P_k\})| \\
&= 2^\alpha(n) + 4^\alpha(2nk - 3n) + 6^\alpha(n) + 9^\alpha(4n) \\
&= n(2^\alpha + (2k - 3)4^\alpha + 6^\alpha + 4.9^\alpha).
\end{aligned}$$

Theorem 5　For $C_n\{P_k\}$ for $k > 1$, the symmetric division index is:

$$SDD(C_n\{P_k\}) = n\left(\frac{20}{3} + 4k\right).$$

Proof　Using the edge partition given in Table 1, we have following computation of symmetric division index:

$$SDD(C_n\{P_k\}) = \sum_{uv \in E(C_n\{P_k\})} \left[\frac{\min(d_u, d_v)}{\max(d_u, d_v)} + \frac{\max(d_u, d_v)}{\min(d_u, d_v)}\right]$$

$$= \frac{5}{2}|E_1(C_n\{P_k\})| + 2|E_2(C_n\{P_k\})| + \frac{13}{6}|E_3(C_n\{P_k\})| + 2|E_4(C_n\{P_k\})|$$

$$= \frac{5}{2}(n) + 2(2nk - 3n) + \frac{13}{6}(n) + 2(4n)$$

$$= n\left(\frac{20}{3} + 4k\right).$$

Theorem 6 For $C_n\{P_k\}$ for $k > 1$, the Atomic bound connectivity index is:

$$ABC(C_n\{P_k\}) = n\left(\frac{2k-1}{\sqrt{2}} + \frac{8}{3}\right).$$

Proof Using the edge partition given in Table 1, we have following computation of symmetric division index:

$$ABC(C_n\{P_k\}) = \sum_{uv \in E(C_n\{P_k\})} \sqrt{\frac{d_u + d_v - 2}{d_u \times d_v}}$$

$$= \sum_{uv \in E_1(C_n\{P_k\})} \sqrt{\frac{d_u + d_v - 2}{d_u \times d_v}} + \sum_{uv \in E_2(C_n\{P_k\})} \sqrt{\frac{d_u + d_v - 2}{d_u \times d_v}} + \sum_{uv \in E_3(C_n\{P_k\})} \sqrt{\frac{d_u + d_v - 2}{d_u \times d_v}} + \sum_{uv \in E_4(C_n\{P_k\})} \sqrt{\frac{d_u + d_v - 2}{d_u \times d_v}}$$

$$= \sum_{uv \in E_1(C_n\{P_k\})} \sqrt{\frac{1+2-2}{1 \times 2}} + \sum_{uv \in E_2(C_n\{P_k\})} \sqrt{\frac{2+2-2}{2 \times 2}} + \sum_{uv \in E_3(C_n\{P_k\})} \sqrt{\frac{2+3-2}{2 \times 3}} + \sum_{uv \in E_4(C_n\{P_k\})} \sqrt{\frac{3+3-2}{3 \times 3}}$$

$$= \frac{1}{\sqrt{2}}|E_1(C_n\{P_k\})| + \frac{1}{\sqrt{2}}|E_2(C_n\{P_k\})| + \frac{1}{\sqrt{2}}|E_3(C_n\{P_k\})| + \frac{2}{3}|E_4(C_n\{P_k\})|$$

$$= n\left(\frac{2k-1}{\sqrt{2}} + \frac{8}{3}\right).$$

Theorem 7 For $C_n\{P_k\}$ for $k > 1$, the geometric arithmetic index is:

$$GA(C_n\{P_k\}) = n\left(2k + \frac{10\sqrt{2} + 6\sqrt{6} + 15}{15}\right).$$

Proof Using the edge partition given in Table 1, we have following computation of Geometric Arithematic index:

$$GA(C_n\{P_k\}) = \sum_{uv \in E(C_n\{P_k\})} \frac{\sqrt{d_u \times d_v}}{\frac{1}{2}(d_u + d_v)}$$

$$= \sum_{uv \in E_1(C_n\{P_k\})} \frac{\sqrt{1 \times 2}}{\frac{1}{2}(1+2)} + \sum_{uv \in E_2(C_n\{P_k\})} \frac{\sqrt{2 \times 2}}{\frac{1}{2}(2+2)} + \sum_{uv \in E_3(C_n\{P_k\})} \frac{\sqrt{2 \times 3}}{\frac{1}{2}(2+3)} + \sum_{uv \in E_4(C_n\{P_k\})} \frac{\sqrt{3 \times 3}}{\frac{1}{2}(3+3)}$$

$$= \frac{2\sqrt{2}}{3}|E_1(C_n\{P_k\})| + 1|E_2(C_n\{P_k\})| + \frac{2\sqrt{6}}{5}|E_3(C_n\{P_k\})| + 1|E_4(C_n\{P_k\})|$$

$$= (n)\frac{2\sqrt{2}}{3} + (2nk - 3n)1 + (n)\frac{2\sqrt{6}}{5} + (4n)1$$

$$= n\left(2k + \frac{10\sqrt{2} + 6\sqrt{6} + 15}{15}\right).$$

Theorem 8 For $C_n\{P_k\}$, modified Randic index is:

$$R'(C_n\{P_k\}) = n(k - n) + \frac{5n}{3}.$$

2.2 Line Graph of the Subdivision Graph of $C_n\{P_k\}$ for $k=1$

The line graph of the subdivision graph of $C_n\{P_k\}$ for $k = 1$ is shown in Figure 2.

The edge partition for $C_n\{P_k\}$ is in Table 2.

Table 2　Edge partition for $C_n\{P_k\}$ for $k=1$

(d_u, d_v) where $uv \in E(C_n\{P_k\})$ for $k = 1$	Number of Edges
(1, 3)	n
(3,3)	$4n$

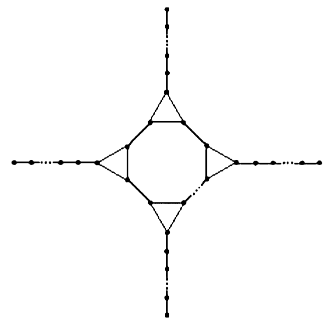

Figure 2　Line Graph of the Subdivision Graph of $C_n\{P_k\}$ for $k=1$

The following results can be obtained immediately from the edge partition of the line graph of the subdivision graph of $C_n\{P_k\}$ for $k = 1$.

Theorem 9　For $C_n\{P_k\}$ for $k = 1$, the Harmonic index is $H(C_n\{P_k\}) = \frac{11n}{6}$.

Theorem 10　For $C_n\{P_k\}$ for $k = 1$, the inverse sum index is $I(C_n\{P_k\}) = \frac{27n}{4}$.

Theorem 11　For $C_n\{P_k\}$ for $k = 1$, the generalized Randic index is $R_\alpha(C_n\{P_k\}) = n\left(\frac{1}{3^\alpha} + \frac{4}{9^\alpha}\right)$.

Theorem 12　For $C_n\{P_k\}$ for $k=1$, the inverse generalized Randic index is $RR_\alpha(C_n\{P_k\}) = n(3^\alpha + 4.9^\alpha)$.

Theorem 13　For $C_n\{P_k\}$ for $k = 1$, the symmetric division index is $SDD(C_n\{P_k\}) = \frac{34n}{3}$.

Theorem 14　For $C_n\{P_k\}$ for $k = 1$, the atomic-bound-connectivity index is $ABC(C_n\{P_k\}) = n\left(\sqrt{\frac{2}{3}} + \frac{8}{3}\right)$.

Theorem 15　For $C_n\{P_k\}$ for $k = 1$, the Geometric arithmetic index is $GA(C_n\{P_k\}) = \frac{n}{2}(\sqrt{3} + 8)$.

Theorem 16　For $C_n\{P_k\}$ for $k = 1$, the modified Randic index is $R'(C_n\{P_k\}) = \frac{5n}{3}$.

3 Conclusions

In this paper, we computed several degree-based topological indices of line graph of subdivision graph of rooted product of cycles with paths. We computed our results with the help of edge partition of based on the degree of end vertices of edges. Our results can be helpful to understand the properties of concerned family of graphs. It is interesting to compute the distance based polynomials and indices for the family of graphs studied in this paper.

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