

第 1 类蜂窝网络的衍生网络逆指数

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摘要: 为进一步探讨第 1 类蜂窝网络的衍生网络, 研究了第 1 类蜂窝网络的衍生网络基于反向度的拓扑指数, 同时计算了蜂窝衍生网络的第 1 和第 2 逆 Zagreb 指数、修正的逆第 2Zagreb 指数、逆对称除法指数、逆 Randic 和反向 Randic 指数、逆反向和指数及反向增强 Zagreb 指数.

关键词: 第 1 类; 蜂窝网络; 度; 逆指数

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Reverse Indices of Derivative Network of Cellular Network Type 1

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Abstract: In order to further study the derivative network of cellular network Type 1, we studied its reversed degree-based topological indices. Meanwhile, we computed the first and second reversed Zagreb indices, reversed modified second Zagreb index, reversed symmetric division index, reversed Randic and inverse Randic index, reverse inverse sum index and reversed augmented Zagreb index.

Key words: type 1; cellular network; degree; reversed index

1 Background Knowledge

In mathematical chemistry, mathematical tools such as polynomials and numbers predict properties of compounds without using quantum mechanics. These tools, in combination, capture information hidden in the symmetry of molecular graphs. Topological indices are numerical parameters of a graph which characterize its topology and are usually graph invariant. They describe the structure of molecules numerically and are used in the development of qualitative structure activity relationships (QSARs). Most commonly known invariants of such kinds are degree-based topological indices. These are actually the numerical values that correlate the structure with various physical properties, chemical reactivity and biological activities. It is an established fact that many properties such as heat of formation, boiling point, strain energy, rigidity and fracture toughness of a molecule are strongly connected to its graphical structure and this fact plays a synergic role in chemical graph theory.

Throughout this report, G is a connected graph, $V(G)$ and $E(G)$ are the vertex set and the edge set respectively and d_v denotes the degree of a vertex v .

The first topological index was introduced by Wiener^[1] and it was named path number, which is now known as Wiener index. In chemical graph theory, this is the most studied molecular topological index due to its wide applications, see for details^[2-3]. Randic index^[4], denoted by $R_{-1/2}(G)$ and introduced by Milan Randic in 1975 is

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also one of the oldest topological index. The Randic index is defined as:

$$R_{-1/2}(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_u d_v}}.$$

In 1998, Bollobas and Erdos^[5] proposed the generalized Randic index and has been studied extensively by both chemist and mathematicians.

The general Randic index is defined as:

$$R_\alpha(G) = \sum_{uv \in E(G)} \frac{1}{(d_u d_v)^\alpha},$$

and the inverse Randic index is defined as $RR_\alpha(G) = \sum_{uv \in E(G)} (d_u d_v)^\alpha$. Obviously $R_{-1/2}(G)$ is the particular case of $R_\alpha(G)$ when $\alpha = -1/2$.

Gutman and Trinajstić introduced first Zagreb index and second Zagreb index, which are defined as: $M_1(G) = \sum_{uv \in E(G)} (d_u + d_v)$ and $M_2(G) = \sum_{uv \in E(G)} (d_u \times d_v)$ respectively. The aim of this paper is to study reverse indices for honey comb derived networks.

The reverse First Zagreb index is defined as:

$$CM_1(G) = \sum_{uv \in CE(G)} c_u + c_v.$$

The reverse Second Zagreb index is defined as:

$$CM_2(G) = \sum_{uv \in CE(G)} c_u \times c_v.$$

The reverse Second Modified Zagreb index is defined as:

$$C^m M_2(G) = \sum_{uv \in CE(G)} \frac{1}{(c_u \times c_v)}.$$

The reverse Symmetric division index is defined as:

$$CSDD(G) = \sum_{uv \in CE(G)} \left\{ \frac{\min(c_u, c_v)}{\max(c_u, c_v)} + \frac{\max(c_u, c_v)}{\min(c_u, c_v)} \right\}.$$

The reverse Harmonic index is defined as:

$$CH(G) = \sum_{uv \in CE(G)} \frac{2}{c_u + c_v}.$$

The reverse Inverse Sum-Index is defined as:

$$CI(G) = \sum_{uv \in CE(G)} \frac{c_u \times c_v}{c_u + c_v}.$$

The reverse Augmented Zagreb Index is defined as:

$$CA(G) = \sum_{uv \in CE(G)} \left\{ \frac{c_u \times c_v}{c_u + c_v - 2} \right\}^3.$$

In this paper, we aim to study the above mentioned reversed degree-based indices for the Honey comb derived network of type 1.

2 Main Results

The Honey comb derived network of type 1 is shown in Figure 1 and is denoted by HCN_1 .

The edge set of $HCN_{1(n)}$ has following five subclasses:

$$\begin{aligned} E_1(HCN_{1(n)}) &= \{uv \in E(HCN_{1(n)}) : d_u = d_v = 3\}; \\ E_2(HCN_{1(n)}) &= \{uv \in E(HCN_{1(n)}) : d_u = 3, d_v = 5\}; \\ E_3(HCN_{1(n)}) &= \{uv \in E(HCN_{1(n)}) : d_u = 3, d_v = 6\}; \end{aligned}$$

$$E_4(HCN_{1(n)1}) = \{uv \in E(HCN_{1(n)1}) : d_u = 5, d_v = 6\};$$

$$E_5(HCN_{1(n)1}) = \{uv \in E(HCN_{1(n)1}) : d_u = d_v = 6\}.$$

The maximum degree of HCN_1 is 6. Hence the reverse edge partition is:

$$CE_1(HCN_{1(n)1}) = \{uv \in E(HCN_{1(n)1}) : c_u = c_v = 4\};$$

$$CE_2(HCN_{1(n)1}) = \{uv \in E(HCN_{1(n)1}) : c_u = 4, c_v = 2\};$$

$$CE_3(HCN_{1(n)1}) = \{uv \in E(HCN_{1(n)1}) : c_u = 4, c_v = 1\};$$

$$CE_4(HCN_{1(n)1}) = \{uv \in E(HCN_{1(n)1}) : c_u = 2, c_v = 1\};$$

$$CE_5(HCN_{1(n)1}) = \{uv \in E(HCN_{1(n)1}) : c_u = c_v = 1\}.$$

Now,

$$|E_1(HCN_{1(n)1})| = |CE_1(HCN_{1(n)1})| = 6;$$

$$|E_2(HCN_{1(n)1})| = |CE_2(HCN_{1(n)1})| = 12(n-1);$$

$$|E_3(HCN_1)| = |CE_3(HCN_1)| = 6n;$$

$$|E_4(HCN_1)| = |CE_4(HCN_1)| = 18(n-1);$$

$$|E_5(HCN_1)| = |CE_5(HCN_1)| = 27n^2 - 57n + 30.$$

For this edge partition, one can compute the following results.

Theorem 1 Let HCN_1 be the honey comb derived network. Then

$$CM_1(HCN_1) = 54n^2 + 12n - 18.$$

Proof From the reverse edge partition given above, we have

$$\begin{aligned} CM_1(HCN_1) &= \sum_{uv \in CE(HCN_1)} (c_u + c_v) \\ &= \sum_{uv \in CE_1(HCN_1)} (c_u + c_v) + \sum_{uv \in CE_2(HCN_1)} (c_u + c_v) + \sum_{uv \in CE_3(HCN_1)} (c_u + c_v) \\ &\quad + \sum_{uv \in CE_4(HCN_1)} (c_u + c_v) + \sum_{uv \in CE_5(HCN_1)} (c_u + c_v) \\ &= |CE_1(HCN_1)|(4+4) + |CE_2(HCN_1)|(4+2) + |CE_3(HCN_1)|(4+1) \\ &\quad + |CE_4(HCN_1)|(2+1) + |CE_5(HCN_1)|(1+1) \\ &= (6 \times 8) + 12(n-1) \times 6 + (6n \times 5) + 18(n-1)3 + (27n^2 - 57n + 30)2 \\ &= 54n^2 + 12n - 18. \end{aligned}$$

Theorem 2 Let HCN_1 be the honey comb derived network. Then

$$CM_2(HCN_1) = 27n^2 + 99n - 6.$$

Proof From the reverse edge partition given above, we have

$$\begin{aligned} CM_2(HCN_1) &= \sum_{uv \in CE(HCN_1)} (c_u \times c_v) \\ &= \sum_{uv \in CE_1(HCN_1)} (c_u \times c_v) + \sum_{uv \in CE_2(HCN_1)} (c_u \times c_v) + \sum_{uv \in CE_3(HCN_1)} (c_u \times c_v) \\ &\quad + \sum_{uv \in CE_4(HCN_1)} (c_u \times c_v) + \sum_{uv \in CE_5(HCN_1)} (c_u \times c_v) \\ &= |CE_1(HCN_1)|(4 \times 4) + |CE_2(HCN_1)|(4 \times 2) + |CE_3(HCN_1)|(4 \times 1) \\ &\quad + |CE_4(HCN_1)|(2 \times 1) + |CE_5(HCN_1)|(1 \times 1) \\ &= (6 \times 16) + 12(n-1) \times 8 + (6n \times 4) + 18(n-1)2 + (27n^2 - 57n + 30)1 \\ &= 27n^2 + 99n - 6. \end{aligned}$$

Theorem 3 Let HCN_1 be the honey comb derived network. Then

$$C^m M_2(HCN_1) = 27n^2 - 54n + \frac{159}{8}.$$

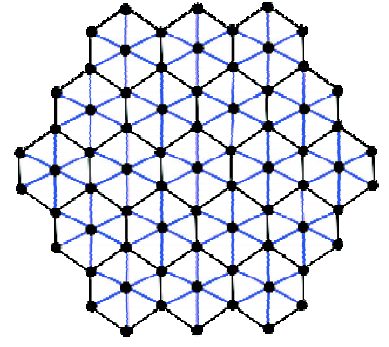


Figure 1 Honey Comb Derived Network of Type 1 for $n=3$

Proof From the reverse edge partition given above, we have

$$\begin{aligned}
 C^m M_2(HCN_1) &= \sum_{uv \in CE(HCN_1)} \frac{1}{(c_u \times c_v)} \\
 &= \sum_{uv \in CE_1(HCN_1)} \left(\frac{1}{c_u \times c_v} \right) + \sum_{uv \in CE_2(HCN_1)} \left(\frac{1}{c_u \times c_v} \right) + \sum_{uv \in CE_3(HCN_1)} \left(\frac{1}{c_u \times c_v} \right) \\
 &\quad + \sum_{uv \in CE_4(HCN_1)} \left(\frac{1}{c_u \times c_v} \right) + \sum_{uv \in CE_5(HCN_1)} \left(\frac{1}{c_u \times c_v} \right) \\
 &= |CE_1(HCN_1)| \left(\frac{1}{4 \times 4} \right) + |CE_2(HCN_1)| \left(\frac{1}{4 \times 2} \right) + |CE_3(HCN_1)| \left(\frac{1}{4 \times 1} \right) \\
 &\quad + |CE_4(HCN_1)| \left(\frac{1}{2 \times 1} \right) + |CE_5(HCN_1)| \left(\frac{1}{1 \times 1} \right) \\
 &= \left(6 \times \frac{1}{16} \right) + 12(n-1) \times \frac{1}{8} + \left(6n \times \frac{1}{4} \right) + 18(n-1) \frac{1}{2} + (27n^2 - 57n + 30) 1 \\
 &= 27n^2 - 54n + \frac{159}{8}.
 \end{aligned}$$

Theorem 4 Let HCN_1 be the honey comb derived network. Then

$$CH(HCN_1) = 27n^2 - \frac{193}{5}n + \frac{31}{2}.$$

Proof From the reverse edge partition given above, we have

$$\begin{aligned}
 CH(HCN_1) &= \sum_{uv \in CE(HCN_1)} \frac{2}{(c_u + c_v)} \\
 &= \sum_{uv \in CE_1(HCN_1)} \left(\frac{2}{c_u + c_v} \right) + \sum_{uv \in CE_2(HCN_1)} \left(\frac{2}{c_u + c_v} \right) + \sum_{uv \in CE_3(HCN_1)} \left(\frac{2}{c_u + c_v} \right) \\
 &\quad + \sum_{uv \in CE_4(HCN_1)} \left(\frac{2}{c_u + c_v} \right) + \sum_{uv \in CE_5(HCN_1)} \left(\frac{2}{c_u + c_v} \right) \\
 &= |CE_1(HCN_1)| \left(\frac{2}{4+4} \right) + |CE_2(HCN_1)| \left(\frac{2}{4+2} \right) + |CE_3(HCN_1)| \left(\frac{2}{4+1} \right) \\
 &\quad + |CE_4(HCN_1)| \left(\frac{2}{2+1} \right) + |CE_5(HCN_1)| \left(\frac{2}{1+1} \right) \\
 &= \left(6 \times \frac{2}{8} \right) + 12(n-1) \times \frac{2}{6} + \left(6n \times \frac{2}{5} \right) + 18(n-1) \frac{2}{3} + (27n^2 - 57n + 30) \frac{2}{2} \\
 &= 27n^2 - \frac{193}{5}n + \frac{31}{2}.
 \end{aligned}$$

Theorem 5 Let HCN_1 be the honey comb derived network. Then

$$CI(HCN_1) = \frac{27}{2}n^2 + \frac{43}{10}n - 1.$$

Proof From the reverse edge partition given above, we have

$$\begin{aligned}
 CI(HCN_1) &= \sum_{uv \in CE(HCN_1)} \left(\frac{c_u c_v}{c_u + c_v} \right) \\
 &= \sum_{uv \in CE_1(HCN_1)} \left(\frac{c_u c_v}{c_u + c_v} \right) + \sum_{uv \in CE_2(HCN_1)} \left(\frac{c_u c_v}{c_u + c_v} \right) + \sum_{uv \in CE_3(HCN_1)} \left(\frac{c_u c_v}{c_u + c_v} \right) \\
 &\quad + \sum_{uv \in CE_4(HCN_1)} \left(\frac{c_u c_v}{c_u + c_v} \right) + \sum_{uv \in CE_5(HCN_1)} \left(\frac{c_u c_v}{c_u + c_v} \right) \\
 &= |CE_1(HCN_1)| \left(\frac{4 \times 4}{4+4} \right) + |CE_2(HCN_1)| \left(\frac{4 \times 2}{4+2} \right) + |CE_3(HCN_1)| \left(\frac{4 \times 1}{4+1} \right) \\
 &\quad + |CE_4(HCN_1)| \left(\frac{2 \times 1}{2+1} \right) + |CE_5(HCN_1)| \left(\frac{1 \times 1}{1+1} \right)
 \end{aligned}$$

$$\begin{aligned}
&= \left(6 \times \frac{16}{8}\right) + 12(n-1) \times \frac{8}{6} + \left(6n \times \frac{4}{5}\right) + 18(n-1) \frac{2}{3} + (27n^2 - 57n + 30) \frac{1}{2} \\
&= \frac{27}{2}n^2 + \frac{43}{10}n - 1.
\end{aligned}$$

Theorem 6 Let HCN_1 be the honey comb derived network. Then

$$CSDD(HCN_1) = 54n^2 - \frac{27}{2}n - 3.$$

Proof From the reverse edge partition given above, we have

$$\begin{aligned}
CSDD(HCN_1) &= \sum_{uv \in CE(HCN_1)} \left\{ \frac{\min(c_u, c_v)}{\max(c_u, c_v)} + \frac{\max(c_u, c_v)}{\min(c_u, c_v)} \right\} \\
&= \sum_{uv \in CE_1(HCN_1)} \left\{ \frac{\min(c_u, c_v)}{\max(c_u, c_v)} + \frac{\max(c_u, c_v)}{\min(c_u, c_v)} \right\} + \sum_{uv \in CE_2(HCN_1)} \left\{ \frac{\min(c_u, c_v)}{\max(c_u, c_v)} + \frac{\max(c_u, c_v)}{\min(c_u, c_v)} \right\} \\
&+ \sum_{uv \in CE_3(HCN_1)} \left\{ \frac{\min(c_u, c_v)}{\max(c_u, c_v)} + \frac{\max(c_u, c_v)}{\min(c_u, c_v)} \right\} + \sum_{uv \in CE_4(HCN_1)} \left\{ \frac{\min(c_u, c_v)}{\max(c_u, c_v)} + \frac{\max(c_u, c_v)}{\min(c_u, c_v)} \right\} \\
&+ \sum_{uv \in CE_5(HCN_1)} \left\{ \frac{\min(c_u, c_v)}{\max(c_u, c_v)} + \frac{\max(c_u, c_v)}{\min(c_u, c_v)} \right\} \\
&= |CE_1(HCN_1)| \left\{ \frac{\min(4, 4)}{\max(4, 4)} + \frac{\max(4, 4)}{\min(4, 4)} \right\} + |CE_2(HCN_1)| \left\{ \frac{\min(4, 2)}{\max(4, 2)} + \frac{\max(4, 2)}{\min(4, 2)} \right\} \\
&+ |CE_3(HCN_1)| \left\{ \frac{\min(4, 1)}{\max(4, 1)} + \frac{\max(4, 1)}{\min(4, 1)} \right\} + |CE_4(HCN_1)| \left\{ \frac{\min(2, 1)}{\max(2, 1)} + \frac{\max(2, 1)}{\min(2, 1)} \right\} \\
&+ |CE_5(HCN_1)| \left\{ \frac{\min(1, 1)}{\max(1, 1)} + \frac{\max(1, 1)}{\min(1, 1)} \right\} \\
&= 6\left(\frac{4}{4} + \frac{4}{4}\right) + 12(n-1) \times \left(\frac{2}{4} + \frac{4}{2}\right) + 6n\left(\frac{1}{4} + \frac{4}{1}\right) + 18(n-1) \times \left(\frac{1}{2} + \frac{2}{1}\right) \\
&+ (27n^2 - 57n + 30) \times \left(\frac{1}{1} + \frac{1}{1}\right) \\
&= 54n^2 - \frac{27}{2}n - 3.
\end{aligned}$$

Theorem 7 Let HCN_1 be the honey comb derived network. Then

$$CR_\alpha(HCN_1) = \frac{27}{1^\alpha}n^2 + \left(\frac{12}{8^\alpha} + \frac{6}{4^\alpha} + \frac{18}{2^\alpha} - \frac{57}{1^\alpha}\right)n + \left(\frac{6}{16^\alpha} - \frac{12}{8^\alpha} + \frac{30}{1^\alpha} - \frac{18}{2^\alpha}\right).$$

Proof From the reverse edge partition given above, we have

$$\begin{aligned}
CR_\alpha(HCN_1) &= \sum_{uv \in CE(HCN_1)} \frac{1}{(c_u \times c_v)^\alpha} \\
&= \sum_{uv \in CE_1(HCN_1)} \left(\frac{1}{(c_u \times c_v)^\alpha}\right) + \sum_{uv \in CE_2(HCN_1)} \left(\frac{1}{(c_u \times c_v)^\alpha}\right) + \sum_{uv \in CE_3(HCN_1)} \left(\frac{1}{(c_u \times c_v)^\alpha}\right) \\
&+ \sum_{uv \in CE_4(HCN_1)} \left(\frac{1}{(c_u \times c_v)^\alpha}\right) + \sum_{uv \in CE_5(HCN_1)} \left(\frac{1}{(c_u \times c_v)^\alpha}\right) \\
&= |CE_1(HCN_1)| \left(\frac{1}{(4 \times 4)^\alpha}\right) + |CE_2(HCN_1)| \left(\frac{1}{(4 \times 2)^\alpha}\right) + |CE_3(HCN_1)| \left(\frac{1}{(4 \times 1)^\alpha}\right) \\
&+ |CE_4(HCN_1)| \left(\frac{1}{(2 \times 1)^\alpha}\right) + |CE_5(HCN_1)| \left(\frac{1}{(1 \times 1)^\alpha}\right) \\
&= \left(6 \times \frac{1}{16^\alpha}\right) + 12(n-1) \times \frac{1}{8^\alpha} + \left(6n \times \frac{1}{4^\alpha}\right) + 18(n-1) \frac{1}{2^\alpha} + (27n^2 - 57n + 30) \frac{1}{1^\alpha} \\
&= \frac{27}{1^\alpha}n^2 + \left(\frac{12}{8^\alpha} + \frac{6}{4^\alpha} + \frac{18}{2^\alpha} - \frac{57}{1^\alpha}\right)n + \left(\frac{6}{16^\alpha} - \frac{12}{8^\alpha} + \frac{30}{1^\alpha} - \frac{18}{2^\alpha}\right).
\end{aligned}$$

Theorem 8 Let HCN_1 be the Honey comb derived network. Then

$$\begin{aligned} CRR_{\alpha}(HCN_1) &= (27 \times 1^{\alpha}) n^2 + (12 \times 8^{\alpha} + 6 \times 4^{\alpha} + 18 \times 2^{\alpha} - 57 \times 1^{\alpha}) n \\ &\quad + (6 \times 16^{\alpha} - 12 \times 8^{\alpha} + 30 \times 1^{\alpha} - 18 \times 2^{\alpha}). \end{aligned}$$

Proof From the reverse edge partition given above, we have

$$\begin{aligned} CRR_{\alpha}(HCN_1) &= \sum_{uv \in CE(HCN_1)} (c_u \times c_v)^{\alpha} \\ &= \sum_{uv \in CE_1(HCN_1)} (c_u \times c_v)^{\alpha} + \sum_{uv \in CE_2(HCN_1)} (c_u \times c_v)^{\alpha} + \sum_{uv \in CE_3(HCN_1)} (c_u \times c_v)^{\alpha} \\ &\quad + \sum_{uv \in CE_4(HCN_1)} (c_u \times c_v)^{\alpha} + \sum_{uv \in CE_5(HCN_1)} (c_u \times c_v)^{\alpha} \\ &= |CE_1(HCN_1)| (4 \times 4)^{\alpha} + |CE_2(HCN_1)| (4 \times 2)^{\alpha} + |CE_3(HCN_1)| (4 \times 1)^{\alpha} \\ &\quad + |CE_4(HCN_1)| (2 \times 1)^{\alpha} + |CE_5(HCN_1)| (1 \times 1)^{\alpha} \\ &= (6 \times 16^{\alpha}) + 12(n-1) \times 8^{\alpha} + (6n \times 4^{\alpha}) + 18(n-1)2^{\alpha} + (27n^2 - 57n + 30)1^{\alpha} \\ &= (27 \times 1^{\alpha}) n^2 + (12 \times 8^{\alpha} + 6 \times 4^{\alpha} + 18 \times 2^{\alpha} - 57 \times 1^{\alpha}) n \\ &\quad + (6 \times 16^{\alpha} - 12 \times 8^{\alpha} + 30 \times 1^{\alpha} - 18 \times 2^{\alpha}). \end{aligned}$$

3 Conclusions

Topological indices are helpful to understand the topology of networks. In this paper, we have studied nine reversed degree-based indices for the Honey comb derived network of type 1. To study the reversed indices for the honey comb derived networks of type 2, type 3 and type 4 is an interesting problem.

[参考文献]

- [1] WIENER H. Structural determination of paraffin boiling points [J]. Journal of the American Chemical Society, 1947, 69 (1): 17–20.
- [2] GAO W, YAN L, SHI L. Generalized Zagreb index of polyomino chains and nanotubes [J]. Optoelectronics and Advanced Materials-Rapid Communications, 2017, 11: 119–124.
- [3] KANG S M, ZAHID M A, NAZEER W, et al. Calculating the degree-based topological indices of dendrimers [J]. Open Chemistry, 2018, 16 (1): 681–688.
- [4] RANDIC M. Characterization of molecular branching [J]. Journal of the American Chemical Society, 1975, 97 (23): 6609–6615.
- [5] BOLLOBAS B, ERDOS P. Graphs of extremal weights [J]. Ars Combinatoria, 1998, 50: 225–233.

