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梯形图线图的一些基于度的拓扑指数

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摘要：图论在电子和电气工程领域中，尤其在信号处理、网络、通信理论等方面起着重要作用. 拓扑指数（TI）是附有图网络的实数，并将化学网络与物理网络联系起来，表示化学性质和化学反应性，其共有 148 种以上的拓扑指数，可是其中没有一个能够完整描述该分子化合物，但它们在某种程度上可以做到这一点. 因此，总是有空间定义新的拓扑索引.

关键词：拓扑指数；梯形图；线图；度
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Some Degree-based Topological Indices for the Line Graph of Ladder Graph

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Abstract：Graph theory has an important role in the field of electronic and electrical engineering，such as in signal processing，networking，communication theory，etc. A topological index（TI）is a real number attached with graph networks and correlates the chemical networks with physical and chemical properties and chemical reactivity. There are more than 148 topological indices but none of them completely describe the molecular compound，but together they do it to some extents. Therefore，there is always space to define new topological indices.
Key words：topological index；ladder graph；line graph；degree

0　Introduction

Let G be a graph having the vertex set $V(G)$ and the edge set $E(G)$. The graph G is called connected set if there exist a connection between all pair of vertices of it. The degree of a vertex u is the number of vertices adjust to it and will be represented by d_u. Throughout this paper，G will represent a connected graph，V its vertex set，E its edge set，and d_v the degree of its vertex v.

In mathematical chemistry，mathematical tools are used to solve problems arising in chemistry. Chemical graph theory is an important area of research in mathematically chemistry which deals with topology of molecular structure such as the mathematical study of isomerism and the development of topological descriptors or indices. TIs are real numbers attached with graph networks and graph of chemical compounds and has applications in quantitative structure-property relationships. TIs remain invariant upto graph isomorphism and help to predict many properties of chemical compounds，networks and nanomaterials，for example，viscosity，boiling points，radius of gyrations，etc without going to lab[1-4].

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Other emerging field is Cheminformatics, which is helpful in guessing biological activity and chemical properties of nanomaterial and networks. In these investigations, some Physico-chemical properties and TIs are utilized to guess the behavior of chemical networks[5-9]. The definitions of known topological indices can be found in [10—12] and references therein.

The line graph $L(G)$ of a simple graph G is obtained by associating a vertex with each edge of the graph and connecting two vertices with an edge iff the corresponding edges of G have a vertex in common. Properties of a graph G that depend only on adjacency between edges may be translated into equivalent properties in $L(G)$ that depend on adjacency between vertices.

In this paper we study the line graph of the Ladder graphs. We computed several degree-based topological indices of understudy families of graphs.

The ladder graph is a planar undirected graph with $2n$ vertices and $3n$-2 edges. The ladder graph can be obtained as the Cartesian product of two path graphs, one of which has only one edge. In this section, let G be the line graph of Ladder Graph. The line graph of ladder graph is given in Figure 1.

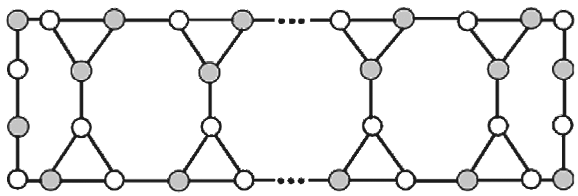

Figure 1 Line Graph of Ladder Graph

1 Methodology

There are three kinds of TIs:

1. Degree-based TIs;
2. Distance-based TIs;
3. Spectral-based TIs.

In this paper, we focus on degree-based TIs. To compute degree-based TIs of line graph of the Ladder graphs, firstly we drawn line graphs and then we divide the edge set of this line graphs into classes based on the degree of the end vertices and compute there cardinality. From this edge partition, we compute our desired results.

2 Main Results

In this section we gave our main results.

Theorem 1 Let G be the line graph of Ladder graph, then we have:

$$SK(G)=\begin{cases}30, & \text{if } n=2,\\ 24n-18, & \text{if } n>2.\end{cases}$$

$$SK_1(G)=\begin{cases}45, & \text{if } n=2,\\ 48n-52, & \text{if } n>2.\end{cases}$$

$$AG_1(G)=\begin{cases}\dfrac{10}{\sqrt{6}}+2+\dfrac{7}{\sqrt{2}}, & \text{if } n=2,\\ \dfrac{10}{\sqrt{6}}+\dfrac{14}{\sqrt{3}}+\dfrac{6n-14}{2}, & \text{if } n>2.\end{cases}$$

$$R'(G)=\begin{cases}3, & \text{if } n=2,\\ \dfrac{9n-1}{6}, & \text{if } n>2.\end{cases}$$

$$SC(G)=\begin{cases}\frac{4}{\sqrt{5}}+\frac{2}{\sqrt{6}}+\frac{4}{\sqrt{7}}, \text{ if } n=2,\\ \frac{4}{\sqrt{5}}+\frac{14}{\sqrt{7}}+\frac{6n-14}{\sqrt{8}}, \text{ if } n>2.\end{cases}$$

Proof　Case 1　$n=2$.

We can divide the edge set of the line graph of ladder graph into following three classes depending on each edge at the end vertices of the degree:

$$E_1(G)=\{e=uv\in E(G);d_u=2 \text{ and } d_v=3\};$$

$$E_2(G)=\{e=uv\in E(G);d_u=3 \text{ and } d_v=3\};$$

$$E_3(G)=\{e=uv\in E(G);d_u=3 \text{ and } d_v=4\}.$$

Now we have $|E_1(G)|=4$, $|E_2(G)|=2$, and $|E_3(G)|=4$.

Case 2　$n>2$.

We can divide the edge set of the line graph of ladder graph into following three classes depending on the degree of end vertices of each edge:

$$E_1(G)=\{e=uv\in E(G);d_u=2,d_v=3\};$$

$$E_2(G)=\{e=uv\in E(G);d_u=3,d_v=4\};$$

$$E_3(G)=\{e=uv\in E(G);d_u=d_v=4\}.$$

Now $|E_1(G)|=4$, $|E_2(G)|=8$, and $|E_3(G)|=6n-14$.

Now by applying definitions and with the help of above edge division, we can compute our desired results.

Following results can also be proved in similar fashion.

Theorem 2　Let G be the line graph ladder graph, then we have:

$$Mrs(G)=\begin{cases}4(2^r3^s+2^s3^r)+2(3^r3^s+3^s3^r)+4(3^r4^s+3^s4^r), \text{ if } n=2,\\ 4(2^r3^s+2^s3^r)+8(3^r4^s+3^s4^r)+(6n-14)(4^r4^s+4^s4^r), \text{ if } n>2.\end{cases}$$

$$ABC(G)=\begin{cases}\frac{4}{\sqrt{2}}+\frac{4}{3}+\frac{2\sqrt{5}}{\sqrt{3}}, \text{ if } n=2,\\ \frac{4}{\sqrt{2}}+\frac{4\sqrt{5}}{\sqrt{3}}+\frac{(6n-14)\sqrt{6}}{4}, \text{ if } n>2.\end{cases}$$

$$GA(G)=\begin{cases}\frac{8\sqrt{6}}{5}+2+\frac{16\sqrt{3}}{7}, \text{ if } n=2,\\ \frac{8\sqrt{6}}{5}+\frac{32\sqrt{3}}{7}+(6n-14), \text{ if } n>2.\end{cases}$$

$$F(G)=\begin{cases}188, \text{ if } n=2,\\ 192n-196, \text{ if } n>2.\end{cases}$$

Theorem 3　Let G be the line graph of ladder Graph, then we have:

$$\mathrm{Re}ZG_1(G)=\begin{cases}\frac{23}{3}, \text{ if } n=2,\\ 3n-1, \text{ if } n>2.\end{cases}$$

$$\mathrm{Re}ZG_2(G)=\begin{cases}\dfrac{513}{35}, & \text{if } n=2,\\ \dfrac{420n-332}{35}, & \text{if } n>2.\end{cases}$$

$$\mathrm{Re}ZG_3(G)=\begin{cases}564, & \text{if } n=2,\\ 768n-1\,000, & \text{if } n>2.\end{cases}$$

Theorem 4 Let G be a line graph of ladder graph, then we have:

$$B_1(G)=\begin{cases}106, & \text{if } n=2,\\ 120n-100, & \text{if } n>2.\end{cases}$$

$$B_2(G)=\begin{cases}278, & \text{if } n=2,\\ 120n-50, & \text{if } n>2.\end{cases}$$

$$HB_1(G)=\begin{cases}1\,020, & \text{if } n=2,\\ 1\,200n-1\,396, & \text{if } n>2.\end{cases}$$

$$HB_2(G)=\begin{cases}3\,544, & \text{if } n=2,\\ 1\,200n-2\,668, & \text{if } n>2.\end{cases}$$

$$BII_1(G)=\begin{cases}12\,636, & \text{if } n=2,\\ (5)^4\times(6)^4\times(8)^8\times(9)^8\times(10)^{6n-14}\times(10)^{6n-14}, & \text{if } n>2.\end{cases}$$

$$BII_2(G)=\begin{cases}(6)^4\times(9)^4\times(12)^2\times(12)^2\times(15)^4\times(20)^4, & \text{if } n=2,\\ (6)^4\times(9)^4\times(15)^8\times(20)^8\times(24)^{6n-14}\times(24)^{6n-14}, & \text{if } n>2.\end{cases}$$

$$HBII_1(G)=\begin{cases}(36)^4\times(81)^4\times(144)^2\times(144)^2\times(225)^4\times(400)^4, & \text{if } n=2,\\ (10)^4\times(36)^4\times(64)^2\times(81)^2\times(100)^{6n-14}\times(100)^{6n-14}, & \text{if } n>2.\end{cases}$$

$$HBII_2(G)=\begin{cases}(36)^4\times(81)^4\times(144)^2\times(144)^2\times(225)^4\times(400)^4, & \text{if } n=2,\\ (36)^4\times(81)^4\times(225)^8\times(400)^8\times(576)^{6n-14}\times(576)^{6n-14}, & \text{if } n>2.\end{cases}$$

Proof Form the information given in theorem 1, we have:

The desired results can be obtained easily with the help of Table 1 and Table 2.

Table 1 Edge Partition of Line Graph of Ladder Graph for $n=2$

$(d_G(u), d_G(v))$	$d_G(e)$	Numbers of edges
(2, 3)	3	4
(3, 3)	4	2
(3, 4)	5	4

Table 2 Edge Partition of Line Graph of Ladder Graph for $n>2$

$(d_G(u), d_G(v))$	$d_G(e)$	Numbers of edges
(2, 3)	3	4
(3, 4)	5	8
(4, 4)	6	$6n-14$

Theorem 5 Let G be the line graph of ladder graph, then we have:

$$H(G,x)=\begin{cases}8x^4+4x^5+8x^6,\\ 4x^4+8x^6+(6n-14)x^7.\end{cases}$$

$$M_1(G,x)=\begin{cases}4x^5+2x^6+4x^7,\\ 4x^5+8x^7+(6n-14)x^8.\end{cases}$$

$$F(G,x)=\begin{cases}4x^{13}+2x^{18}+4x^{25},\\ 4x^{13}+8x^{25}+(6n-14)x^{32}.\end{cases}$$

Theorem 6 Let G is the wheel graph of ladder graph, then we have:

$$ABCII(G)=\begin{cases}8x^4+4x^5+8x^6, & \text{if } n=2,\\ 4x^4+8x^6+(6n-14)x^7, & \text{if } n>2.\end{cases}$$

$$GAII(G)=\begin{cases}\left(\frac{2\sqrt{6}}{5}\right)^4\times\left(\frac{4\sqrt{3}}{7}\right)^4, & \text{if } n=2,\\ \left(\frac{2\sqrt{6}}{5}\right)^4\times\left(\frac{4\sqrt{3}}{7}\right)^8, & \text{if } n>2.\end{cases}$$

$$MZ_1^{\alpha}(G)=\begin{cases}(5^{\alpha})^4\times(6^{\alpha})^2\times(7^{\alpha})^4, & \text{if } n=2,\\ (5^{\alpha})^4\times(7^{\alpha})^8\times(8^{\alpha})^{6n-14}, & \text{if } n>2.\end{cases}$$

3 Conclusion

In present report, we computed several degree based topological indices of line graph of ladder graph. During the last two decades a large number of numerical graph invariants (topological indices) have been defined and used for correlation analysis in theoretical chemistry, pharmacology, toxicology, and environmental chemistry. Topological indices are used to guess properties of chemical compounds without going to wet lab. Almost all properties of a chemical compound can be obtained from the topological indices. In this way, our results are important for chemists and drug designers.

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